

9 Orthogonality

1. Consider the following TRS:

$$\begin{array}{lcl} f(g(x), b) & \rightarrow & f(g(x), d) \\ g(c) & \rightarrow & g(a) \\ a & \rightarrow & c \\ b & \rightarrow & d \end{array}$$

- (a) What is (are) the critical pair(s)?
(b) Show that the TRS is confluent.
2. For each of the following systems, determine the critical pairs. Which of these systems is confluent? Motivate your answer!

- (a) $f(f(x)) \rightarrow a$
(b) $f(x, 0) \rightarrow x, \quad f(x, s(y)) \rightarrow s(f(f(x, y), y))$
(c) $s(p(x)) \rightarrow x, \quad p(s(x)) \rightarrow x$
(d) $f(g(x), y) \rightarrow x, \quad g(a) \rightarrow b$
(e) $a(a(x)) \rightarrow a(b(a(x)))$
(f) $or(x, y) \rightarrow x, \quad or(x, y) \rightarrow y$
(g) $a(x) \rightarrow a(b(x)), \quad b(c(x)) \rightarrow b(x), \quad a(c(x)) \rightarrow a(b(x))$

3. Reduce the CL term $SII(IK)(KK(II))$ to normal form with each the following reduction strategies:

- (a) leftmost outermost,
(b) leftmost innermost,
(c) parallel outermost,
(d) parallel innermost,
(e) full substitution.