

8 Dependency pairs

1. Consider the TRS with rules:

$$\begin{array}{lll} \text{append}(\text{nil}, z) & \rightarrow & z \\ \text{append}(\text{cons}(x, y), z) & \rightarrow & \text{cons}(x, \text{append}(y, z)) \\ \text{rev}(\text{nil}) & \rightarrow & \text{nil} \\ \text{rev}(\text{cons}(x, y)) & \rightarrow & \text{append}(\text{rev}(y), \text{singular}(x)) \\ \text{singular}(x) & \rightarrow & \text{cons}(x, \text{nil}) \end{array}$$

(Here, as before, x, y, z are variables; the other symbols are function symbols.)

- (a) Identify the constructors and the defined symbols of this TRS.
- (b) Give the dependency pairs for this TRS.
- (c) Prove termination of this TRS.

2. What are the dependency pairs of the following TRS?

$$\begin{array}{lll} \text{f}(\text{a}(x), y) & \rightarrow & \text{a}(\text{f}(x, \text{g}(y))) \\ \text{g}(x) & \rightarrow & \text{a}(x) \end{array}$$

3. Prove termination of the following rules using dependency pairs

$$\begin{array}{lll} \text{f}(\text{h}(x), y) & \rightarrow & \text{g}(x, \text{f}(x, \text{h}(y))) \\ \text{g}(x, \text{h}(y)) & \rightarrow & \text{g}(\text{h}(x), y) \end{array}$$

4. Consider the following TRS.

$$\begin{array}{lll} \text{f}(\text{a}(x), y) & \rightarrow & \text{f}(\text{b}(y), x) \\ \text{g}(\text{a}(x), y) & \rightarrow & \text{h}(\text{b}(y), x) \\ \text{g}(\text{b}(x), y) & \rightarrow & y \\ \text{h}(x, \text{a}(y)) & \rightarrow & \text{g}(\text{a}(y), \text{a}(\text{i}(x))) \\ \text{h}(x, \text{b}(y)) & \rightarrow & \text{a}(x) \\ \text{i}(\text{a}(x)) & \rightarrow & \text{i}(x) \end{array}$$

- (a) Identify the dependency pairs of this TRS.
- (b) Apply the weakly monotonic interpretations processor to simplify the immediate DP problem.
Do not use the graph!
- (c) Prove termination using the graph and subterm criterion.