

5 Confluence, unification and critical pairs

1. Give a TRS and give in that TRS three terms, where in the first we have to parallel redexes, in the second we have two nested redexes, in the third we have two overlapping redexes.
2. Compute first overlaps and then the critical pairs of the following systems. Then argue whether they are WCR or not.

(a) $f(x) \rightarrow a \quad f(x) \rightarrow g(f(x)) \quad g(f(x)) \rightarrow f(h(x))$

(b) $f(f(a)) \rightarrow a$

(c) $f(x, 0) \rightarrow x \quad f(x, s(y)) \rightarrow s(f(f(x, y), y))$

(d) $s(p(x)) \rightarrow x \quad p(s(x)) \rightarrow x$

(e) $f(g(x), y) \rightarrow x \quad g(a) \rightarrow b$

(f) $a(a(x)) \rightarrow a(b(a(x)))$

(g) $or(x, y) \rightarrow x \quad or(x, y) \rightarrow y$

(h) $a(x) \rightarrow a(b(x)) \quad b(c(x)) \rightarrow b(x) \quad a(c(x)) \rightarrow a(b(x))$

3. Compute the critical pairs for the following TRSs:

$$\begin{array}{lcl} f(x, g(x)) & \rightarrow & a \\ f(g(x), x) & \rightarrow & b \end{array}$$

7 Equational reasoning and completion

1. From [2].

Consider the following equational specification:

$$f(f(f(x))) \approx g(x)$$

$$g(g(g(x))) \approx f(x)$$

- (a) Use completion to find a confluent and terminating TRS for this equational system. Hint: start with the rule for the first equation in the length increasing form, and for readability write strings instead of terms.
- (b) Is the equation $g(g(x)) \approx x$ derivable in the specification?

2. From [4] [Day 2 exercise 11].

Complete the TRS consisting of the following two rewrite rules:

$$\begin{array}{lcl} f(f(x)) & \rightarrow & g(x) \\ f(g(f(x))) & \rightarrow & f(x) \end{array}$$

3. From [6].

Show that the completion algorithm diverges if applied to $f(g(f(x))) \rightarrow g(f(x))$.

4. From [4] [Day 3, exercise 2]. Consider the ES consisting of one equation:

$$f(f(x)) \approx g(x)$$

Give three different complete TRSs ‘computing’ this ES.